Output Regulation of Discrete-Time Piecewise-Linear Systems With Application to Controlling Chaos

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***Abstract*—This paper presents an approach to output regulation of discrete-time piecewise-linear systems. A sufficient condition to guarantee output regulation of such systems via state feedback has been obtained based on piecewise-quadratic Lyapunov functions. It is shown that the output regulation controller can be obtained by solving a set of linear-matrix inequalities. Application to controlling chaos is also given to illustrate the performance of the proposed approach.**

***Index Terms*—Controller design, discrete-time systems, linear-matrix inequality (LMI), output regulation, piecewise-linear systems.**

I. Introduction

PIECEWISE-LINEAR systems have been a topic of research interests in the circuits, systems, and control community for many years, see, for example, [1]–[10]. It is known that piecewise-linear systems arise often in practical control systems when piecewise-linear components are encountered. These components include dead-zone, saturation, relays, and hysteresis among many others. It is also known that many chaotic circuits can be constructed by piecewise-linear circuits [3], [4]. Moreover, many nonlinear systems can also be approximated by the piecewise-linear systems, at least in the operating regions of the systems, and thus the piecewise-linear systems provide a powerful means of analysis and design for more general nonlinear control systems.

A number of significant results have been reported on analysis and controller design of such piecewise-linear systems during the last few years. For example, the authors in [5] studied a basic issue, that is, the well-posedness of piecewise-linear systems. Necessary and sufficient conditions for bimodal systems to be well-posed have been derived, and the extension to the multimodal case has also been discussed. The authors in [6], [7] presented results on stability and optimal performance analysis for piecewise-linear systems based on a piecewise-continuous Lyapunov function. The authors in [8] presented a number of results on stability analysis, controller design, H_∞ analysis, and H_∞ controller design for the piecewise-linear systems based on a piecewise Lyapunov function. Recently, we proposed a method of stability analysis and a number of controller design approaches for piecewise-linear systems with affine terms by using novel piecewise Lyapunov functions [9]–[11]. Relevant works on stability analysis of hybrid systems and neural networks with piecewise-linear functions have also been reported in [12]–[15].

However, to our best knowledge, there are few reports on the output regulation of piecewise-linear systems in the open literature though there are plenty of results on output regulation of linear systems and also nonlinear systems [16]–[21]. In fact, the output regulation of linear systems has been thoroughly solved in the seventies, please see [16], [17] for references. It has been shown that the existence of a solution to the problem of output regulation can be determined by two linear-matrix equalities (LMIs). Correspondingly, the problem of output regulation of nonlinear systems has been addressed by the well-known Isidori and Byrnes equations [18]–[20], which are partial differential equations and might be difficult to solve for general nonlinear systems. Some approximating approaches have been developed to avoid the difficulty in solving the Isidori & Byrnes equations by using neural networks [21]. However, the approach only leads to approximate regulation due to the approximation of Isidori and Byrnes equations.

In this paper, we will present an approach to output regulation of the discrete-time piecewise-linear systems. Two sufficient conditions will be given to guarantee solution to the problem of output regulation via state feedback for such piecewise-linear systems based on piecewise-quadratic Lyapunov functions.

The rest of the paper is organized as follows. Section II introduces the piecewise-linear system model and the stability theorems. Section III presents results of output regulation for such systems based on state feedback. Application to controlling chaos is presented in Section IV, which is followed by conclusions in Section V.

II. Problem Formulation

Consider piecewise-discrete-time linear systems of the form

$$\begin{aligned} x(t+1) &= A_l x(t) + B_l u(t) + E_l w(t) \\ w(t+1) &= S_l w(t) \\ e(t) &= C_l x(t) + D_l w(t), \qquad \text{for } x \in X_l, l \in L \end{aligned} \tag{2.1}$$

where $\{X_l\}_{l\in L} \subseteq \Re^n$ denotes a partition of the state space into a number of closed polyhedral subspaces, L is the index set of subspaces, $x(t) \in \Re^n$ the system state variables, $u(t) \in \Re^m$ the system control input, $w(t) \in \Re^q$ the exogenous reference and disturbance input, $e(t) \in \Re^m$ the error to be regulated. It is noted that the second equation is often called the exosystem.

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For the definition of state trajectory and solution to the piecewise-linear system (2.1), please refer to [5]–[7] for details. Here we assume that given any initial condition $x(0) = x_0$, the difference (2.1) has a unique solution for all $t > 0$. We also assume that when the state of the system transits from the region X_l to X_j at the time t, the dynamics of the system is governed by the dynamics of the local model of X_l at that time. For future use, we also define a set Ω that represents all possible transitions from one region to another, that is

$$\Omega := \{l, j \,|\, x(t) \in X_l, x(t+1) \in X_j, j \neq l\}.$$

Remark 2.1: It is easily seen that the affine terms appearing in a normal piecewise-linear system model [6], [7], [11] can be lumped into the last term of the first equation of (2.1).

The objective of the control design is to find a control law such that the closed-loop system by setting $w = 0$ is asymptotically stable and the output of the closed-loop system tracks the desired reference for any initial conditions, that is, $e(t) \to 0$ as $t \to \infty$. This problem is often called the *output regulation problem*.

The following assumptions are made in this paper:

A1) the eigenvalues of $S_l, l \in L$ are on or outside of the unit circle;

A2) the pair (A_l, B_l) is stabilizable in the Lyapunov stability sense;

It is well known that for continuous time systems when $L = \{1\}$ and $X_1 = \Re^n$, complete solutions to the above mentioned output regulation problem were established in [12], [13]. These solutions can be adapted for discrete-time systems when $L = \{1\}$ and $X_1 = \Re^n$ as follows.

Proposition 2.1: Under the assumptions A1-A2 with $L = \{1\}$ and $X_1 = \Re^n$, the problem of output regulation is solvable via state feedback if and only if: 1) there exist matrices Π and Γ that satisfy the following linear-matrix equations:

$$\begin{aligned} \Pi S_1 &= A_1\Pi + B_1\Gamma + E_1 \\ C_1\Pi + D_1 &= 0 \end{aligned} \tag{2.2}$$

or equivalently, 2) the matrix

$$\begin{bmatrix} A_1 - \lambda I & B_1 \\ C_1 & 0 \end{bmatrix} \tag{2.3}$$

is nonsingular for each λ which is an eigenvalue of S_1.

It should be pointed out that when a piecewise-linear system is considered, the stability condition is quite different from that for a linear time-invariant system. The following stability result based on a piecewise-quadratic Lyapunov function [6], [7], [11], is adopted here.

Proposition 2.2: The piecewise-linear system (2.1) with $u \equiv w \equiv 0$ is globally exponentially stable if the following LMIs admit a set of positive definite matrix solutions P_l:

$$A_l^T P_l A_l - P_l < 0, \qquad l \in L \tag{2.4}$$
$$A_l^T P_j A_l - P_l < 0, \qquad l, j \in \omega. \tag{2.5}$$

One natural question is whether the output regulation problem is solvable for piecewise-linear systems and what the conditions are. In the next section, we will address the output regulation problem for these piecewise-linear systems.

III. Output Regulation Via State Feedback

Consider a piecewise-linear system described by (2.1) and a piecewise state-feedback control law

$$u(t) = K_l x(t) + G_l w(t), \qquad x \in X_l, \qquad l \in L. \tag{3.1}$$

The closed-loop system can be obtained by substituting (3.1) into (2.1) leading to the following equations:

$$\begin{aligned} x(t+1) &= (A_l + B_l K_l)x(t) + (E_l + B_l G_l)w(t) \\ w(t+1) &= S_l w(t) \\ e(t) &= C_l x(t) + D_l w(t), \qquad \text{for } x \in X_l, l \in L. \end{aligned} \tag{3.2}$$

Then, we are ready to present the following solution to the output regulation problem via state feedback.

Theorem 3.1: Under the assumption A1, the problem of output regulation is solvable via state feedback if the following are true.

a) There exist matrices Π_l and Γ_l that satisfy the following linear-matrix equations:

$$\Pi_l S_l = A_l \Pi_l + B_l \Gamma_l + E_l, \qquad l \in L \tag{3.3}$$
$$C_l \Pi_l + D_l = 0, \qquad l \in L \tag{3.4}$$

with boundary conditions

$$(\Pi_l - \Pi_j) S_l = 0, \qquad l, j \in \Omega. \tag{3.5}$$

b) The following LMIs admit a set of positive definite matrix P_l and a set of matrices Q_l:

$$0 > \begin{bmatrix} -P_l & P_l A_l^T + Q_l^T B_l^T \\ A_l P_l + B_l Q_l & -P_l \end{bmatrix}, \qquad l \in L \tag{3.6}$$
$$0 > \begin{bmatrix} -P_l & P_l A_l^T + Q_l^T B_l^T \\ A_l P_l + B_l Q_l & -P_j \end{bmatrix}, \qquad l, j \in \Omega. \tag{3.7}$$

Moreover, the controller gain for each local subsystem is given by

$$K_l = Q_l P_l^{-1}, \qquad l \in L \tag{3.8}$$
$$G_l = \Gamma_l - K_l \Pi_l, \qquad l \in L. \tag{3.9}$$

Proof: At first, we show the internal stability of the closed-loop system with state feedback gains given by (3.8)–(3.9). Set $w = 0$ in (3.2) and it can be easily shown via the Schur complement that (3.6) and (3.7) respectively together with (3.8) imply

$$(A_l + B_l K_l)^T P_l^{-1} (A_l + B_l K_l) - P_l^{-1} < 0, \qquad l \in L$$

and

$$(A_l + B_l K_l)^T P_j^{-1} (A_l + B_l K_l) - P_l^{-1} < 0, \qquad l, j \in \Omega.$$

Thus, it follows from Proposition 2.3 that the closed-loop system is globally exponentially stable.

We now show that the output of the closed-loop system tracks the desired reference for any initial conditions, that is, $e(t) \to 0$ as $e(t) \to \infty$ as $t \to \infty$. Define $z_l = x - \Pi_l w$ for $x \in X_l$, one has for $x \in X_l$

$$\begin{aligned} z_l(t+1) &= x(t+1) - \Pi_l w(t+1) \\ &= (A_l + B_l K_l)x(t) + (E_l + B_l G_l)w(t) - \Pi_l S_l w(t) \\ &= (A_l + B_l K_l)x(t) + (E_l + B_l G_l)w(t) \\ &\quad - (A_l \Pi_l + B_l \Gamma_l + E_l)w(t) \\ &= (A_l + B_l K_l)[(x(t) - \Pi_l w(t)] \\ &\quad + B_l K_l \Pi_l w(t) \\ &\quad + B_l G_l w(t) - B_l \Gamma_l w(t) \\ &= (A_l + B_l K_l) z_l(t) + B_l(K_l \Pi_l + G_l - \Gamma_l) w(t) \\ &= (A_l + B_l K_l) z_l(t) \end{aligned} \tag{3.10}$$

where (3.3) and (3.9) have been used. It follows from (3.10) that when the state stays in one region,

$$\begin{aligned} &z_l(t+k) = (A_l + B_l K_l)^k z_l(t), \\ &\qquad \{x(t), \dots x(t+k-1)\} \in X_l, \qquad l \in L \end{aligned} \tag{3.11}$$

which implies that

$$\begin{aligned} &\|z_l(t+k)\| \le r_l^k \|z_l(t)\|, \\ &\qquad \{x(t), \dots x(t+k-1)\} \in X_l, \qquad l \in L \end{aligned} \tag{3.12}$$

where r_l is defined to be the spectral radius of the matrix $(A_l + B_l K_l)$. Then, we consider the state transition from one region to another. Suppose, without loss of generality, that the state of the system transits from the region l to the region j at time t, one then has

$$\begin{aligned} &z_j(t+1) \\ &= x(t+1) - \Pi_j w(t+1) \\ &= (A_l + B_l K_l)x(t) + (E_l + B_l G_l)w(t) - \Pi_j S_l w(t) \\ &= (A_l + B_l K_l)x(t) + (E_l + B_l G_l)w(t) \\ &\quad - \Pi_l S_l w(t) + \Pi_l S_l w(t) - \Pi_j S_l w(t) \\ &= (A_l + B_l K_l)x(t) + (E_l + B_l G_l)w(t) \\ &\quad - (A_l \Pi_l + B_l \Gamma_l + E_l)w(t) + (\Pi_l - \Pi_j) S_l w(t) \\ &= (A_l + B_l K_l)[(x(t) - \Pi_l w(t)] + B_l K_l \Pi_l w(t) \\ &\quad + B_l G_l w(t) - B_l \Gamma_l w(t) + (\Pi_l - \Pi_j) S_l w(t) \\ &= (A_l + B_l K_l) z_l(t) + B_l(K_l \Pi_l + G_l - \Gamma_l) w(t) \\ &\quad + (\Pi_l - \Pi_j) S_l w(t) \\ &= (A_l + B_l K_l) z_l(t). \end{aligned} \tag{3.13}$$

This implies that

$$\begin{aligned} &\|z_j(t+1)\| \le (r_l) z_l(t), \\ &\qquad \mathbf{x}(t) \in X_l, x(t+1) \in X_j, \qquad l, j \in \Omega. \end{aligned} \tag{3.14}$$

Then combining (3.12) and (3.14) leads to that for any initial conditions and any starting region, say l_0

$$\|z_j(t)\| \le (\max_{i \in L} r_i)^t z_{l_0}(0) \tag{3.15}$$

which implies that $z_j(t) \to 0$ for any j as $t \to \infty$. Furthermore, one has

$$\begin{aligned} e(t) &= C_l x(t) + D_l w(t) \\ &= C_l z_l + (C_l \Pi_l + D_l) w(t) \\ &= C_l z_l. \end{aligned} \tag{3.16}$$

It then follows from (3.16) and (3.15) that

$$\lim_{t \to \infty} e(t) = 0 \tag{3.17}$$

and the proof is thus completed. $\Delta\Delta\Delta$

If the same matrix Π for all the subregions is chosen, then we have the following corollary.

Corollary 3.1: Under the assumption A1, the problem of output regulation is solvable via state feedback if the following are true.

a) There exist matrices Π and Γ_l that satisfy the following linear-matrix equations:

$$\Pi S_l = A_l \Pi + B_l \Gamma_l + E_l, \qquad l \in L \tag{3.18}$$

$$C_l \Pi + D_l = 0, \qquad l \in L. \tag{3.19}$$

b) The following LMIs admit a set of positive definite matrix P_l and a set of matrices Q_l

$$0 > \begin{bmatrix} -P_l & P_l A_l^T + Q_l^T B_l^T \\ A_l P_l + B_l Q_l & -P_l \end{bmatrix}, \qquad l \in L \tag{3.20}$$

$$0 > \begin{bmatrix} -P_l & P_l A_l^T + Q_l^T B_l^T \\ A_l P_l + B_l Q_l & -P_j \end{bmatrix}, \qquad l, j \in \Omega. \tag{3.21}$$

Moreover, the controller gain for each local subsystem is given by

$$K_l = Q_l P_l^{-1}, \qquad l \in L, \tag{3.22}$$

$$G_l = \Gamma_l - K_l \Pi, \qquad l \in L. \tag{3.23}$$

IV. Application to Controlling Chaos

We will apply the proposed approach to controlling chaos in this section. Consider the well-known Chua's circuit [22], which is shown in Fig. 1. By applying the Kirchhoff's law, the dynamical behavior of Chua's circuit is described as follows:

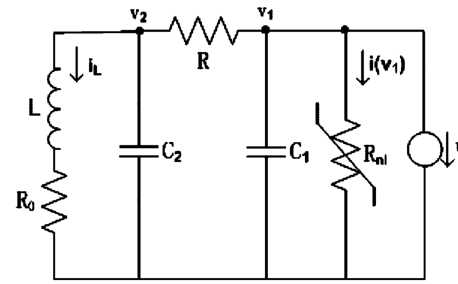

Fig. 1. Chua's circuit.

$$\begin{cases} C_1 \frac{dv_1}{dt} = \frac{v_2 - v_1}{R} - g(v_1) - u \\ C_2 \frac{dv_2}{dt} = \frac{v_1 - v_2}{R} - i_L \\ L \frac{di_L}{dt} = v_2 + R_0 i_L \end{cases} \tag{4.1}$$

where v_1 and v_2 are the voltages at the ends of C_1 and C_2, respectively, i_L is the current in the inductance L,, and u is the current from generator as active control action of the circuit.

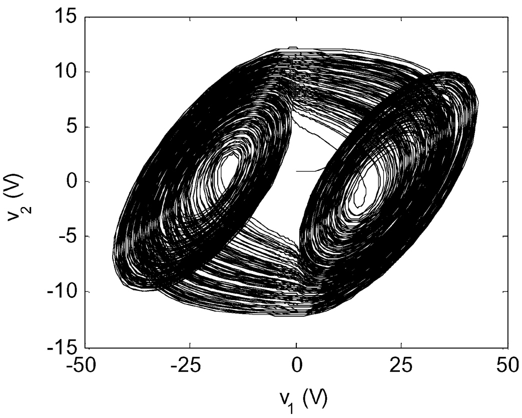

Fig. 2. Chaotic attractor of Chua's circuit.

The characteristic of the nonlinear resistor $g(v_1)$ is a piecewise-linear function defined by

$$g(v_1) = G_b v_1 + 0.5(G_a - G_b) \times (|v_1 + E| - |v_1 - E|) \tag{4.2}$$

where $G_a, G_b < 0$. Assuming $v_1 \in [-d, d], d > E > 0$, (4.2) can be formulated as

$$g(v_1) = \begin{cases} G_b v_1 + (G_a - G_b)E, & v_1 \geq E \\ G_a v_1, & -E < v_1 < E \\ G_b v_1 - (G_a - G_b)E, & v_1 \leq -E. \end{cases} \tag{4.3}$$

Choose $R = 10/7, R_0 = 0, C_1 = 1/9, C_2 = 1, L = 1/7, G_b = -0.5, G_a = -4.5, E = 1, d = 45$. A chaotic behavior can be observed as shown in Fig. 2 when the active current generator is absent and the initial conditions of the circuit are $v_1(0) = 0, v_2(0) = 1$, and $i_L(0) = 0$. Let $x_1 = v_1, x_2 = v_2$, and $x_3 = i_L$. Then, with sampling period $T = 0.01s$ one has the following discrete-time piecewise-linear model of the controlled Chua's circuit:

$$\begin{aligned} x(t+1) &= A_l x(t) + B_l u(t) + a_l \\ y(t) &= C_l x(t) \\ l &= 1, 2, 3 \end{aligned} \tag{4.4}$$

and the corresponding three sets are defined by $\{l = 1 \text{ if } x_1 \geq E\}, \{l = 2 \text{ if } |x_1| < E\}$ and $\{l = 3 \text{ if } x_1 \leq -E\}$, respectively. The system matrices are given by

$$A_1 = \begin{bmatrix} 0.982\,38 & 0.062\,215 & -0.000\,312\,38 \\ 0.006\,912\,8 & 0.992\,89 & -0.009\,964\,6 \\ 0.000\,242\,96 & 0.069\,753 & 0.999\,65 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1.408 & 0.074\,864 & -0.000\,353\,4 \\ 0.008\,318\,2 & 0.992\,92 & -0.009\,964\,7 \\ 0.000\,274\,86 & 0.069\,753 & 0.999\,65 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0.982\,38 & 0.062\,215 & -0.000\,312\,38 \\ 0.006\,912\,8 & 0.992\,89 & -0.009\,964\,6 \\ 0.000\,242\,96 & 0.069\,753 & 0.999\,65 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -0.089\,201 \\ -0.000\,312\,38 \\ -7.3042\text{e-}006 \end{bmatrix}$$

$$a_1 = \begin{bmatrix} 0.356\,81 \\ 0.001\,249\,5 \\ 2.9217\text{e-}005 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} -0.107\,31 \\ -0.000\,353\,4 \\ -8.0101\text{e-}006 \end{bmatrix}$$

$$a_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} -0.089\,201 \\ -0.000\,312\,38 \\ -7.3042\text{e-}006 \end{bmatrix}$$

$$a_3 = \begin{bmatrix} -0.356\,81 \\ -0.001\,249\,5 \\ -2.9217\text{e-}005 \end{bmatrix}$$

$$C_1 = C_2 = C_3 = [1 \quad 0 \quad 0].$$

The control objective is to design a state feedback controller so that the output of the closed-loop system would track a sinusoidal function $y_d = A \sin 2\pi f t T$ with $A = 10, T = 0.01$ s, and $f = 0.1$ Hz. With consideration of the constant affine term a_l and the tracking purpose, we introduce the following exosystem:

$$w(t+1) = Sw(t), \quad w(0) = [1, 0, A]^T \tag{4.5}$$

where

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\pi f T & \sin 2\pi f T \\ 0 & -\sin 2\pi f T & \cos 2\pi f T \end{bmatrix}, \quad w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \end{bmatrix}.$$

It is noted that $w_1(t) = 1$ and $w_2(t) = A \sin 2\pi f t T$. With this exosystem, the first equation of system (4.4) can be described as

$$x(t+1) = A_l x(t) + B_l u(t) + E_l w(t) \tag{4.6}$$

where

$$E_1 = \begin{bmatrix} 0.356\,81 & 0 & 0 \\ 0.001\,249\,5 & 0 & 0 \\ 2.9217\text{e-}005 & 0 & 0 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} -0.356\,81 & 0 & 0 \\ -0.001\,249\,5 & 0 & 0 \\ -2.9217\text{e-}005 & 0 & 0 \end{bmatrix}.$$

One thus can define the error equation as

$$e(t) = x_1(t) - w_2(t).$$

By solving (3.3)–(3.4) and (3.6)–(3.7), the following solutions based on piecewise-quadratic Lyapunov functions are obtained:

$$P_1 = \begin{bmatrix} 77.887 & -12.601 & -9.4407 \\ -12.601 & 15.638 & -8.6913 \\ -9.4407 & -8.6913 & 87.852 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 77.886 & -12.615 & -9.4402 \\ -12.615 & 15.638 & -8.6912 \\ -9.4402 & -8.6912 & 87.852 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 77.887 & -12.601 & -9.4407 \\ -12.601 & 15.638 & -8.6913 \\ -9.4407 & -8.6913 & 87.852 \end{bmatrix}$$

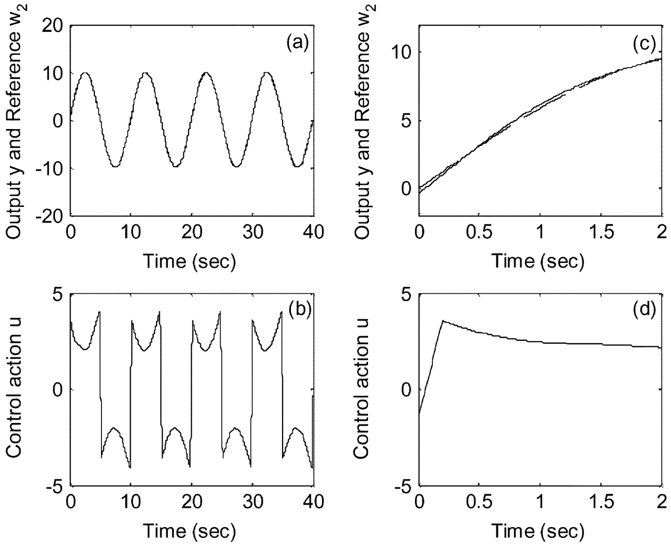

Fig. 3. Response of the closed-loop chaotic control system.

$$K_1 = [11.132 \quad 11.251 \quad 2.1432]$$
$$K_2 = [13.222 \quad 9.4637 \quad 1.7807]$$
$$K_3 = [11.132 \quad 11.251 \quad 2.1432]$$
$$G_1 = [4 \quad -12.961 \quad -0.664\,54]$$
$$G_2 = [0 \quad -10.776 \quad -0.5517]$$
$$G_3 = [-4 \quad -12.961 \quad -0.664\,54].$$

The simulations have been conducted with the obtained controller and Fig. 3 shows the responses of the closed-loop control system with initial condition $x(0) = [0\ 1\ 0]^T$, where (a) and (b) are for the whole simulation period of 40 s; (c) and (d) for the initial 2 seconds. It can be easily observed that the tracking has been well achieved and the transient performance is satisfactory.

V. Conclusion

In this paper, an approach to output regulation is developed for discrete-time piecewise-linear systems based on piecewise-quadratic Lyapunov functions. It is shown that the output regulation controller can be determined by solving a set of LMIs. Application to controlling chaos is presented to demonstrate the performance of the proposed approach. One interesting topic of future research is whether the proposed approach can be applied to piecewise-linear systems with unmodeled dynamics and/or parameter uncertainties. It appears that the dynamic output feedback and/or adaptive approaches might be needed.

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